The efficient modeling of blade lean effects within the turbomachinery throughflow method

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A derivation of an additional body force term representing blade lean effects in the radial equilibrium equation is presented, and the implementation of the extended equation in turbine throughflow calculations is described. The force includes a dissipative component associated with blade viscous losses complying with Horlock's recommendations on consistent modeling of losses. The required body force is evaluated entirely within the throughflow method, and auxiliary blade-to-blade calculations are not required. The effect of 15° of stator blade lean is determined for a single-stage transonic test problem. Comparisons with three-dimensional flow computations made with an Euler solver show good agreement, and the results demonstrate a significant influence of lean on the flow field at rotor inlet.

Keywords: turbines; blade lean; throughflow; streamline curvature

Introduction

There is increasing interest in the use of circumferential blade lean in the final stages of l.p. steam turbines. Leaning stator blades in the direction of rotation affords the designer an additional degree of freedom in controlling the hub pressure and the incidence and relative Mach number of the flow incident to the rotor. Fully three-dimensional (3-D) methods in which the blade surfaces are completely specified; e.g., Denton[1] and Dawes[2] include blade lean effects automatically. These methods require extensive data input and long computation times and are best used sparingly for detailed evaluation of individual blade rows or stages. Turbine design requires consideration of the interaction between multiple stages and the evaluation of a variety of geometrical proposals; consequently axisymmetric throughflow methods retain an important role in the theoretical prediction of the flow field.

The streamline curvature throughflow program SLEQ (Denton[3]) is in extensive use for theoretical studies of steam turbines. The flow is modeled as being axisymmetric and primarily inviscid with the flow equations given in the streamline curvature formulation. Discrete increases of entropy determined from loss correlations are imposed at calculation stations to represent viscous losses. In the original form of the program blade forces did not appear explicitly in the radial equilibrium equation (R.E.E.). Flow swirl angles were prescribed within blade passages and at trailing edges, thereby implying tangential and axial forces, while radial forces were simply neglected. When blade surfaces are not everywhere parallel with radii, they inevitably contribute some net radial force, which becomes particularly significant if a substantial degree of blade lean is present. Within an axisymmetric formulation this can best be introduced as a body force acting on the fluid which contributes an additional term in the radial equilibrium equation. The imposed entropy increases imply a drag force which also contributes a radial component. As pointed out by Horlock,[4] this implied drag force should be parallel with the flow direction if the loss model is to be consistent.

Bosman and Marsh[5] derived a Poisson equation for the axisymmetric stream function incorporating blade lean effects and consistent loss modeling as an extension of the matrix throughflow method. A derivation of a radial equilibrium equation with the corresponding extensions for the streamline curvature formulation of the throughflow method is given in the first section of this paper. The R.E.E. obtained is shown to conform with that given previously by Wennerstrom.[6] The derivation is included for the purposes of review and clarity, in a notation complying with current conventions for specifying blade geometry and flow vectors. Although the extended form of the R.E.E. has been known for more than a decade, there are, to the authors' knowledge, no reports in the literature of its implementation in throughflow computations except where body forces have been obtained via blade to blade calculations (e.g., Novak and Hearsey[7] and Jennions and Stow[8]). By contrast, in the present treatment, the evaluation of the body force giving the blade lean effect forms an integral part of the throughflow calculation and is performed to an accuracy comparable with the other aspects of the method.

In the succeeding sections results obtained with the extended R.E.E. included in throughflow calculations for a single turbine stage with stator blade lean are presented. Comparison is then made with results from the 3-D inviscid time marching program STAGE3D of Denton[1] in order to assess the success of the body force representation.

Geometrical specification of a leaned blade

The "quasi-orthogonal" mesh lines (q.o.'s) containing calculation stations at entry and exit planes of a blade row are defined by the line intersection of the meridional (r-z) plane with conical surface containing the complete set of blade edges within the annulus. The q.o.'s are defined with zero lean for simplicity, since, given the assumption of axisymmetry, there is no advantage in choosing them coincident with leaned blade edges. The angle between the q.o. direction q and the radial direction in the meridional plane is defined as the sweep angle ε. It is convenient to specify blade lean by the angle δ_t between the projection of the trailing edge into the r-θ plane and the radial direction. That is, δ_t refers to the trailing edge as "seen" from downstream and is measured positive in the direction of rotation. The lean δ of the intersection of the blade surface with the r-θ plane differs slightly from δ_t when the q.o. is swept. It can be deduced from Figure 1(a) that

$$\tan\delta=\tan\delta_t-\tan\alpha_0\tan\varepsilon$$

The specification of a blade in a throughflow calculation requires q.o.'s at leading and trailing edges and one or more intermediate blade passage q.o.'s. Provision is made for each q.o. to have individual values of sweep and blade lean, the treatment of leading edge and passage q.o.'s being similar to that just described.

The blade-to-blade geometry is specified in the axial tangential (z-θ) plane. For a straight-backed blade the blade surface angle α_0 is calculated from the opening/pitch ratio (i.e., $\alpha_0=\cos^{-1} o/s$). Three-dimensional calculations indicated that α_0 changed very little with 15° variation in lean angle; hence it was assumed to be independent of lean in the test problem.

Further geometrical considerations regarding the effect of blade lean on the flow velocity vector are contained in Equation 13.

Derivation of the radial equilibrium equation including body force terms

The radial equilibrium equation employed in throughflow programs has been quoted in various forms following the initial derivations given by Smith[9] and Novak.[10] We shall use as a basis for the derivation of the R.E.E. the formulation of the Euler equations given by Novak and Hearsey[11] and include body forces to represent blade effects in the manner of Jennions and Stow.[8] The effect of blade surface pressures will be absorbed into a body force vector **F** having components F_z, F_r. F_θ. A subdivision of **F** into conservative and dissipative components is necessary if a consistent loss model as discussed by Horlock[4] is to be realized. The inviscid axial, radial, and tangential equations read

$$\frac{1}{\rho}\frac{\partial p}{\partial z}=-C_m\frac{\partial C_z}{\partial m}+F_z \tag{1}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial r}=\frac{C_\theta^2}{r}-C_m\frac{\partial C_r}{\partial m}+F_r \tag{2}$$

$$0=-\frac{C_m}{r}\frac{\partial}{\partial m}(rC_\theta)+F_\theta \tag{3}$$

For a quasi-orthogonal with sweep ε we have the transformation

$$\frac{\partial}{\partial q}=\cos\varepsilon\frac{\partial}{\partial r}+\sin\varepsilon\frac{\partial}{\partial z} \tag{4}$$

where q denotes distance measured along the q.o. When q.o.'s are chosen along blade leading and trailing edges they define flow conditions over conical surfaces of rotation, given that axisymmetry is assumed. Combining Equations 1 and 2 with the transformation 4 gives

$$\frac{1}{\rho}\frac{\partial p}{\partial q}=\frac{C_\theta^2}{r}\cos\varepsilon-C_m\frac{\partial C_r}{\partial m}\cos\varepsilon-C_m\frac{\partial C_z}{\partial m}\sin\varepsilon+F_r\cos\varepsilon+F_z\sin\varepsilon \tag{5}$$

Now

$$C_r=C_m\sin\phi,\quad C_z=C_m\cos\phi,\quad \frac{1}{r_m}=\frac{\partial\phi}{\partial m}$$

where r_m is the streamlines meridional radius of curvature. After substitution and some rearrangement we obtain for the radial equilibrium equation

$$\frac{1}{\rho}\frac{\partial p}{\partial q}=\frac{C_\theta^2}{r}\cos\varepsilon-\frac{C_m^2}{r_m}\cos(\phi+\varepsilon)-C_m\frac{\partial C_m}{\partial m}\sin(\phi+\varepsilon)$$
$$+F_r\cos\varepsilon+F_z\sin\varepsilon \tag{6}$$

Let us split the body force vector **F** into subvectors **N** and **D** acting in directions normal to and tangential to the blade surface:

$$\mathbf{F}=\mathbf{N}+\mathbf{D}$$

N results from the blade surface pressure distributions, while **D** is a dissipative force representing surface shear forces. It can

Notation

C	Gas velocity
D	Dissipative component of blade force acting on the fluid
F	Body force vector
h, h_0	Enthalpy, total enthalpy
I	Rothalpy
i, j, k	Unit vectors in axial, tangential, and radial directions
$\mathbf{l}_1, \mathbf{l}_2$	Unit vectors defining the blade surface
m	Meridional coordinate
N	Normal component of blade force
p	Static pressure
q	Quasi-orthogonal coordinate
r	Radius
r_m	Radius of curvature
s	Entropy
T	Static temperature
V	Gas velocity relative to rotor
z	Axial coordinate
α	Flow swirl angle measured from the axial direction defined positive for swirl in the direction of blade rotation
$\bar{\alpha}$	Flow swirl angle in the stream surface plane
δ	q.o. lean angle measured positive in the direction of rotation
ε	q.o. sweep angle between radial and q.o. directions in the meridional plane, measured positive when the outer end of the q.o. lies downstream of the hub/q.o. intercept
ϕ	Flow pitch angle in the meridional plane
ρ	Density
ω	Angular velocity of rotor

Subscripts

z, θ, r	Indicate axial, tangential, and radial components
m, q	Indicate meridional and quasi-orthogonal components
t	Value describing blade trailing edge

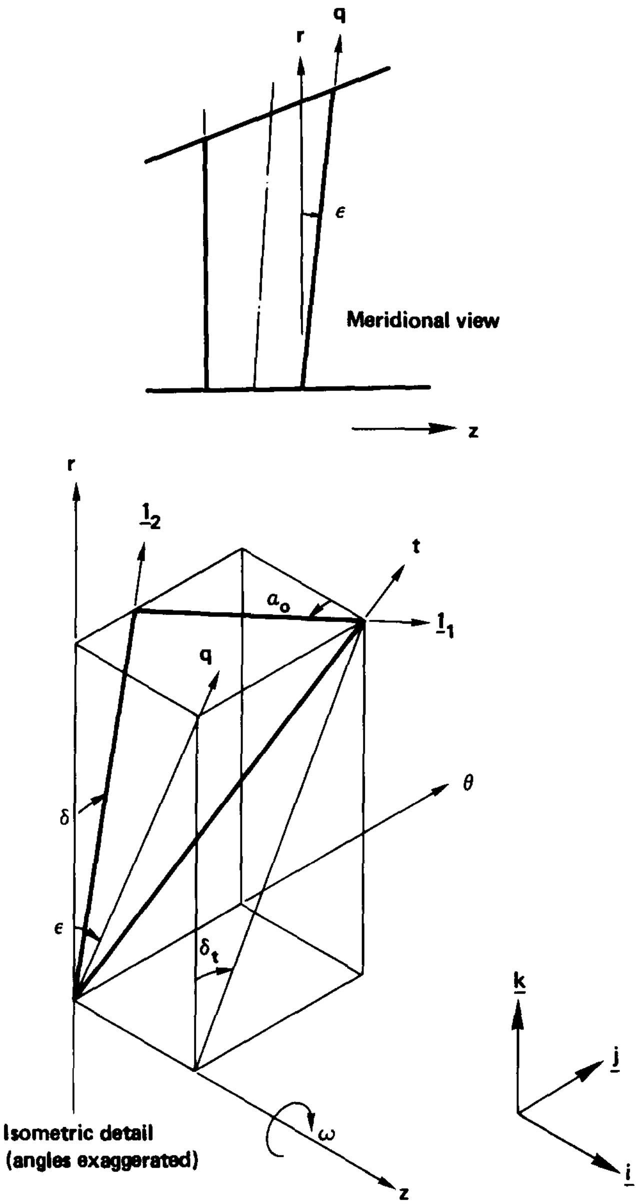

Figure 1(a) Quasi-orthogonal and trailing edge geometry for a leaned blade

be seen in Figure 1(a) that the vectors

$\mathbf{l}_1 = \mathbf{i}\cos\alpha_0 + \mathbf{j}\sin\alpha_0$

$\mathbf{l}_2 = \mathbf{j}\sin\delta + \mathbf{k}\cos\delta$

are tangential to the blade surface; hence the normal force vector must satisfy

$\mathbf{N}\cdot\mathbf{l}_1 = 0, \qquad \mathbf{N}\cdot\mathbf{l}_2 = 0$

i.e.,

$$N_z = -N_\theta \tan\alpha_0, \qquad N_r = -N_\theta \tan\delta \tag{7}$$

The drag vector **D** is opposed to the velocity vector **C**, so resolving vectors in the axial, tangential and radial directions with the aid of Equation 7 and Figure 2 we have

$$F_z = -N_\theta \tan\alpha_0 - D\cos\bar{\alpha}\cos\phi \tag{8}$$

$$F_\theta = N_\theta - D\sin\bar{a} \tag{9}$$

$$F_r = -N_\theta \tan\delta - D\cos\bar{\alpha}\sin\phi \tag{10}$$

It is convenient to eliminate N_θ from the component equations and express N_z and N_r in terms of F_θ and D, since F_θ is known from the overall change in angular momentum (Equation 3), and D from Horlock's consistent loss arguments (Equation 16).

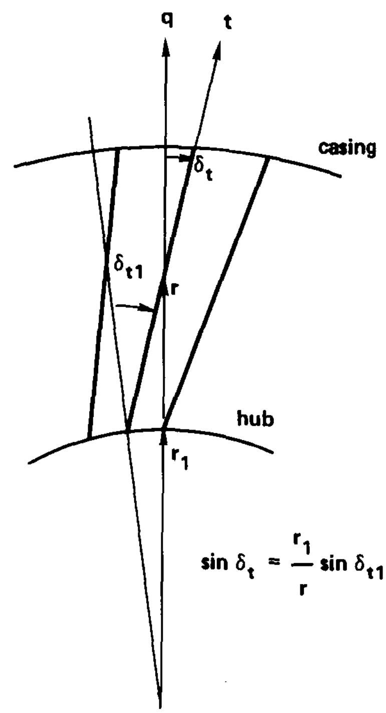

Figure 1(b) Axial view from downstream of leaned blade trailing edges

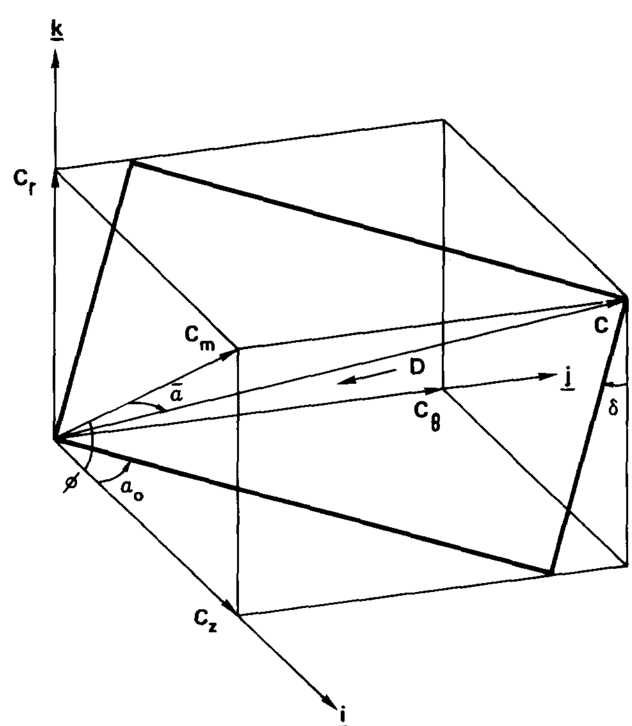

Figure 2 Velocity components and dissipative force *D* lying in a leaned blade surface

Multiplying Equation 9 by $\tan \alpha_0$ and $\tan \delta$ and summing with Equations 8 and 10, respectively, gives

$$F_z = -F_\theta \tan \alpha_0 - D(\sin \bar{\alpha} \tan \alpha_0 + \cos \bar{\alpha} \cos \phi) \quad (11)$$

$$F_r = -F_\theta \tan \delta - D(\sin \bar{\alpha} \tan \delta + \cos \bar{\alpha} \sin \phi) \quad (12)$$

Inspection of the geometry of flow vectors in Figure 2 reveals that swirl angles $\bar{\alpha}$ in the stream surface are related to angles α_0 in the blade-to-blade plane by the relation

$$\tan \bar{\alpha} = \tan \alpha_0 \cos \phi + \sin \phi \tan \delta \quad (13)$$

The meridional force F_m consistent with specified losses is given by Gallimore[12] for the zero lean case. Resolving in the meridional direction, we have

$$F_m = F_z \cos \phi + F_r \sin \phi$$

Substituting from Equations 11 and 12 and setting $\delta = 0$, we obtain

$$F_m = -F_\theta \tan \bar{\alpha} - D/\cos \bar{\alpha} \quad (14)$$

This agrees with Gallimore's result except for an apparent typographical error in the sign of the first term on the RHS. The body force component in the quasi-orthogonal direction required in the radial equilibrium equation (6) is given by

$$F_q = F_r \cos \varepsilon + F_z \sin \varepsilon$$

Substituting for F_r and F_z from Equations 11 and 12 and recalling the geometrical relation between δ and δ_t given in the preceding section, we find that

$$F_q = -F_\theta \tan \delta_t \cos \varepsilon - D[\sin \bar{\alpha} \tan \delta_t \cos \varepsilon + \cos \bar{\alpha} \sin(\phi + \varepsilon)] \quad (15)$$

The consistent loss argument (Horlock[4]) relates the dissipative force D to the meridional entropy gradient

$$D = T \frac{\partial s}{\partial m} \cos \bar{\alpha} \quad (16)$$

Substituting for F_θ and D from Equations 3 and 16, we have

$$F_q = -\frac{C_m}{r} \frac{\partial}{\partial m}(rC_\theta) \tan \delta_t \cos \varepsilon - T \frac{\partial s}{\partial m}[\sin \bar{\alpha} \cos \bar{\alpha} \tan \delta_t \cos \varepsilon + \cos^2 \bar{\alpha} \sin(\phi + \varepsilon)] \quad (17)$$

Reformulating Equation 6 in terms of the derivative of meridional velocity by utilizing the relationships

$$T \frac{\partial s}{\partial q} = \frac{\partial h}{\partial q} - \frac{1}{\rho} \frac{\partial p}{\partial q} \quad \text{and} \quad h_0 = h + \frac{1}{2}(C_\theta^2 + C_m^2)$$

yields the final working form of the R.E.E.:

$$\frac{1}{2} \frac{\partial}{\partial q}(C_m^2) = \frac{\partial h_0}{\partial q} - T \frac{\partial s}{\partial q} - \frac{1}{2r^2} \frac{\partial}{\partial q}(r^2 C_\theta^2) + \frac{C_m^2}{r_m} \cos(\phi + \varepsilon) + C_m \frac{\partial C_m}{\partial m} \sin(\phi + \varepsilon) - F_q \quad (18)$$

Equations 17 and 18 are equivalent to Equation 30 of Wennerstrom.[6] Wennerstrom's sign convention for lean and sweep angles is the reverse of that adopted here, and he gives the rotating coordinate form. This can be obtained from Equation 18 by making the substitutions

$$C_m = V_m, \qquad C_\theta = V_\theta + \omega r$$

$$I = h + \frac{V^2}{2} - \frac{\omega^2 r^2}{2}$$

Implementation of the revised R.E.E.

The meridional derivative of angular momentum appearing in the R.E.E. represents the tangential component of blade forces (see Equation 3) and is calculated via flow directions imposed in the axial tangential (blade-to-blade) plane at each blade passage or trailing edge q.o. In throughflow analysis rather coarse grids are usually employed, so it is necessary to consider the numerical representation of the meridional derivative with some care. In order to preserve consistency in the integral sense, upwind differences of rC_θ are used. This ensures that at each q.o. a force is imposed accounting for the total tangential flow deflection incurred up to that station.

Imposed flow swirl angles are calculated from camber lines for passage q.o.'s. At trailing edges exit swirl angles are determined from a correlation with opening-to-pitch ratio. The corresponding angles in the stream surface are given by Equation 13, and a supersonic deviation is included in regions where the blade row is choked. These stream surface swirl angles determine the circumferential velocity components via the relation

$$C_\theta = C_m \tan \bar{\alpha}$$

where $\bar{\alpha}$ is the angle between the resultant velocity and meridional direction. Supersonic deviation angles are calculated using an extension of the method derived by Scholz.[13] An allowance for profile losses is made in determining the choking flow, and the equation of momentum in the blade back direction is utilized. Shock losses for a specified exit Mach number are given by the method, just as in the analytical oblique shock and Rankine-Hugoniot relations in gas dynamics.

Results and discussion

In order to assess the success of the body force representation described, calculations were made for a hypothetical turbine stage with both throughflow and the fully three-dimensional Euler solver STAGE3D (Denton[1]). It was considered important to compute the complete final stage field in order to separate the effects of lean at the stator trailing edge from the imposed downstream boundary conditions. Stator lean angles of 0° and 15° were considered. The small inherent lean of the rotor, due to blade twist, was neglected in the throughflow calculations. For the latter the stator blade was described by q.o.'s at leading and trailing edges and one or more in the blade passage. Schematic meridional geometry and stator blade profiles for the stage are shown in Figure 3.

Entry conditions and the downstream reference pressure for both the 3-D and throughflow calculations were identical, and the fluid was assumed to be a perfect gas with a specific heat ratio of 1.32. A routine for circumferentially averaging the 3-D results was appended to STAGE3D. The mean flow required to conserve the integrated mass flow, total enthalpy, and impulse function was calculated. The entropy of the mean flow was then found, and the corresponding numerical loss coefficient calculated.

The effect of stator blade lean on the streamlines of the throughflow calculation is shown in Figure 4, in this case with two passage q.o.'s. The action of the radial force deflecting the streamlines toward the hub within the blade row is clearly visible. When only a single passage q.o. at mid axial chord was specified, the deflection was delayed until the trailing edge, but the distributions of flow parameters and downstream pattern differed very little from the case shown.

We have made a point of calculating the radial blade force terms in the throughflow R.E.E. via upwind differencing.

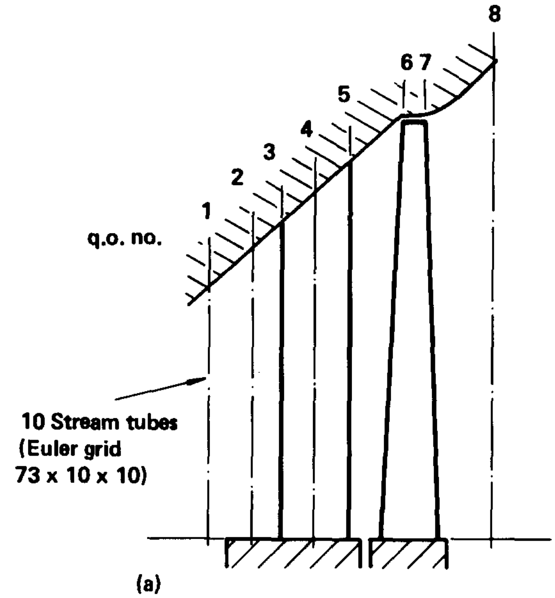

Figure 3(a) Throughflow meridional geometry

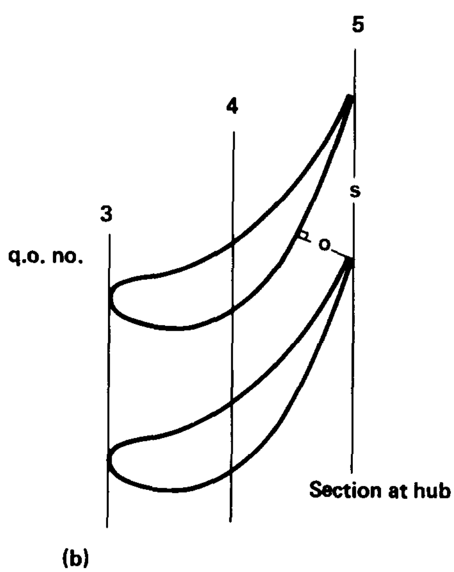

Figure 3(b) Stator blade passage and profile shapes

Calculations were made with alternative forms of differencing for comparison. As an example, for the case with two passage q.o.'s, centered differencing was found to diminish by 36% the change in hub Mach number resulting from lean. For greater numbers of passage q.o.'s the difference between these alternatives will be less important, but for coarse q.o. spacings upwind differencing showed better agreement with the Mach number profiles obtained from 3-D Euler calculations. The flow distributions which follow were obtained with a single passage q.o.

The radial distributions of Mach number and pressure at stator exit calculated by SLEQ are shown in Figures 5 and 6 together with circumferentially averaged results from STAGE3D for stator lean angles of 0° and 15°. The changes in the hub and tip Mach numbers due to lean calculated by both methods are in good agreement. In the throughflow calculations the maximum Mach number which occurred at the hub was significantly reduced from 1.55 to 1.27 by the inclusion of the blade lean, though the Mach number levels were about 5% higher than in the averaged 3-D results.

The STAGE3D calculations, although nominally inviscid, were found to have stator losses up to 10% in magnitude arising from numerical viscosity implicit in the method and from aerodynamic shocks in the hub region. The profile losses calculated from the correlation of Balje and Binsley[14] employed in SLEQ were about 5%, and this accounts to a large extent for the differences in Mach number at the stator exit. This was confirmed by a numerical experiment in which the Mach number profiles were forced into close coincidence by imposing the STAGE3D loss levels in the SLEQ throughflow calculation.

Novak and Hearsey,[7] using blade-to-blade calculations in conjunction with the R.E.E., reported only a minor influence of blade lean on root reaction ratio. Figure 6 shows that a significant influence of lean on hub pressure was obtained in the present study. The hub conditions were supersonic rather than subsonic as in Novak's case, and this probably accounts for the size of the effect. A considerable part of the appeal of the method described here lies in the fact that a hybrid combination of throughflow and blade-to-blade calculations is not required.

Table 1 shows the hub Mach numbers and overall mass flows G obtained with both calculation methods. The methods show very good mass flow agreement for the leaned case, while the 3-D calculation was slightly lower (1.4%) for the zero lean geometry. These differences are, perhaps, less than one would expect to result from the difference in loss levels between the calculations. This suggests that stream surface twist due to

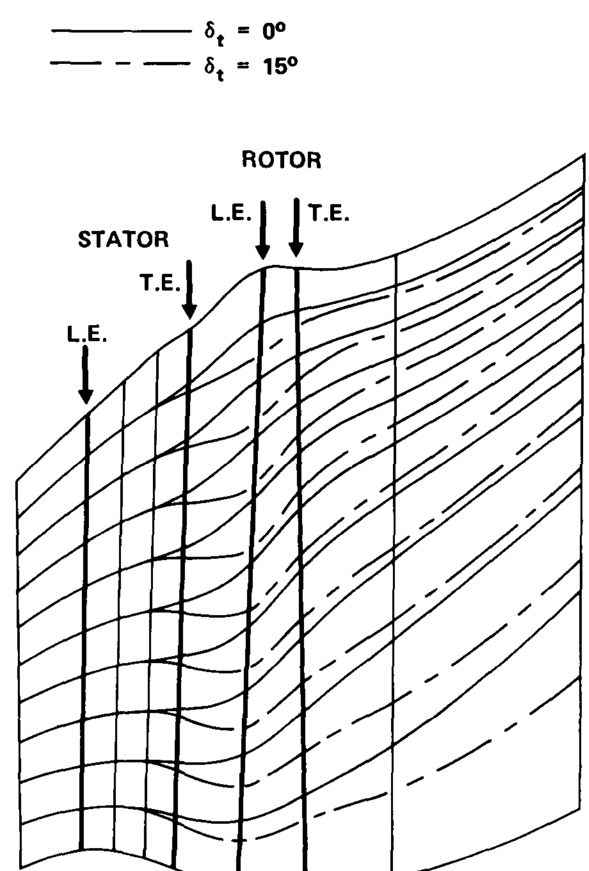

Figure 4 Streamline positions computed by the throughflow program showing the effect of lean

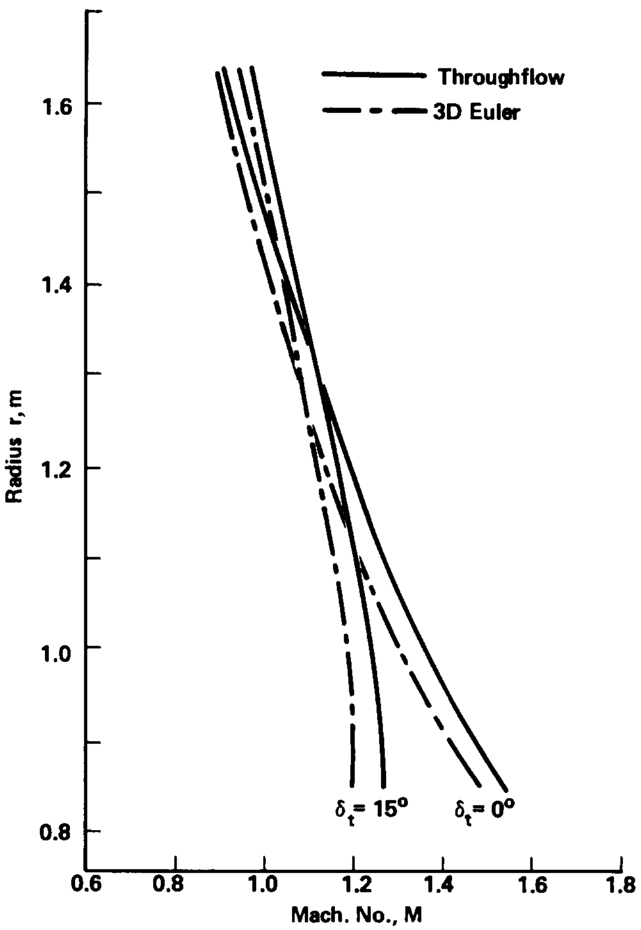

Figure 5 Radial distributions of Mach number at stator blade trailing edge

nonaxisymmetry is also a significant influence. The effect of dissipative terms in the R.E.E. was isolated in a numerical experiment. Their effect was found to be very minor in the test problem, since they contributed a change of less than 0.005 in stator exit Mach number at any radial location. They may, however, be significant in off-design conditions or in any application where q.o.'s are more closely aligned with the stream direction.

The radial distributions of flow swirl angles at the stator trailing edge are shown in Figure 7. For the leaned blades the agreement between throughflow and Euler programs is very good, exhibiting a small supersonic deviation from the subsonic correlation angle $\cos^{-1} o/s$. In the zero lean case, however, the throughflow calculation shows supersonic deviation reaching 7° at the hub, whereas the 3-D Euler calculation showed supersonic deviation initially rising with Mach number but, somewhat surprisingly, falling away to zero at the hub.

The numerical viscosity in the Euler solver mentioned previously would be expected to generate some secondary flow opposing supersonic deviation. The effect would be strongest at the hub where it is reinforced by the radial pressure gradient. The presence of lean reduces radial pressure gradient, and the velocities reached are generally lower; this may account for the better agreement found for the lean case. The computed distributions of absolute rotor exit velocity (Figure 8(a)) are in very good agreement for both cases. The effect of lean has been to increase velocity and, hence, leaving loss over the outer half of the annulus.

The associated exit swirl distributions (Figure 8(b)) again show very good agreement between the methods for the lean case. There is, however, a significant discrepancy in the hub region for zero lean. The 3-D calculation shows a hub swirl of almost 50° compared with 26° from throughflow. This difference has been traced to a substantially reduced magnitude of relative velocity in the 3-D calculation. The relative flow angles differ slightly, the 3-D result showing 3° overturning probably due to secondary flow effects. The lower relative velocity is thought to result from increased numerical losses associated with a higher relative inlet Mach number to the rotor blade.

An investigation of 3-D flow features arising from viscosity would require the use of a full Navier Stokes solver such as that reported by Dawes.[2] Overall the agreement between the throughflow and 3-D Euler computations is good and is very good in the lean case, confirming the body force representation of lean in the throughflow method.

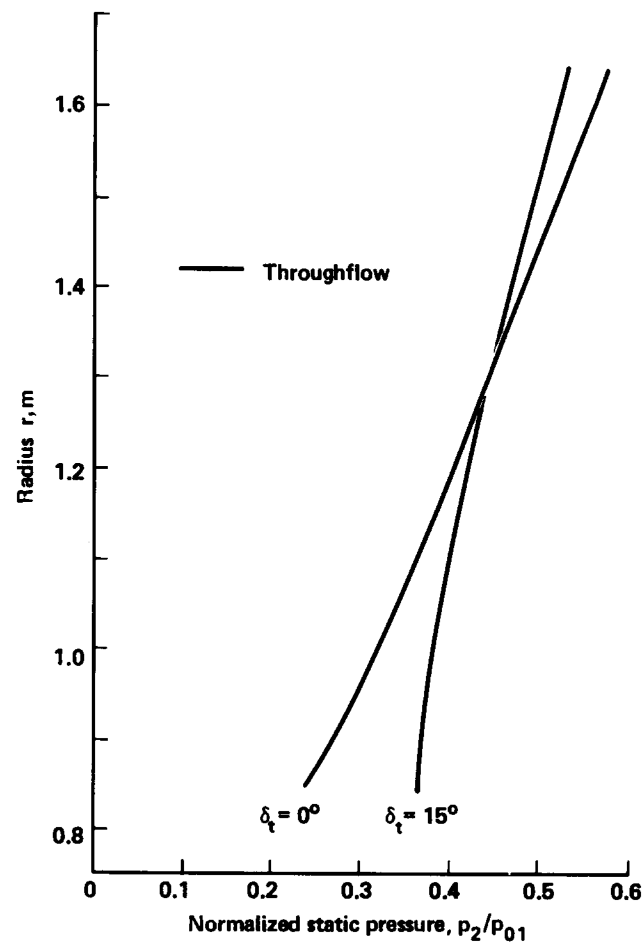

Figure 6 Radial distributions of static pressure at stator blade trailing edge

Table 1 Computed mass flows and hub Mach numbers

	Throughflow		3-D Solver	
δ_t	G	M_{hub}	G	M_{hub}
0	36.07	1.55	35.55	1.49
15	36.07	1.27	35.98	1.20

Conclusions

It is possible to represent blade lean within axisymmetric turbine throughflow calculations by introducing the radial component of a body force which is determined via the blade flow deflection correlations intrinsic to the method. The sensitivity of the pressure and Mach number profiles to lean angle predicted by this method appear very plausible, and the results obtained for the leaned case compare well with calculations made with a fully 3-D time marching program.

Further work to minimize extraneous effects from numerical viscosity in the time-marching calculations is required to provide a more precise comparison of throughflow and 3-D methods.

Stator blade lean can have a significant influence on root reaction and the relative inlet velocity field to the rotor.

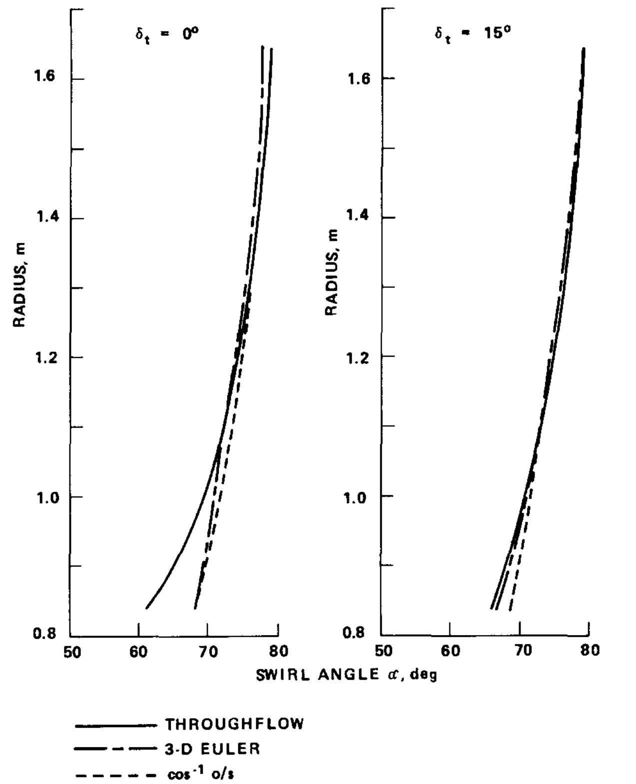

Figure 7 Radial distributions of flow swirl angles at stator blade trailing edge

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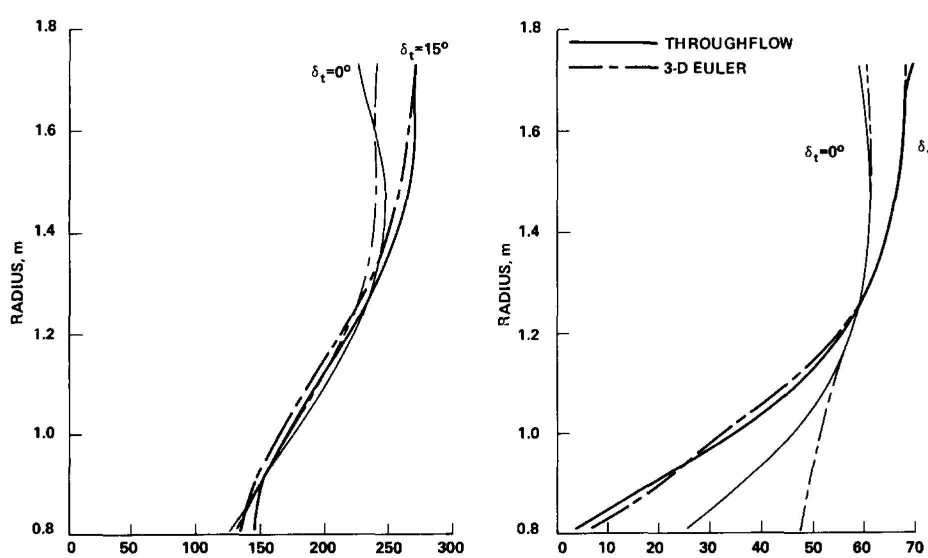

Figure 8 Radial distributions of absolute velocity and swirl angle at rotor trailing edge

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